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Supersymmetric gauge theories from branes and orientifold six-planes

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Abstract: We study brane configurations in the presence of orientifold six-planes. After deriving the curves for $\mathcal{N} = 2$ supersymmetric $SU(N_c)$ gauge theories with one flavor in the symmetric or antisymmetric representation and N_f fundamental flavors, we rotate the brane configuration, reducing the supersymmetry to $\mathcal{N} = 1$. For the case of an antisymmetric flavor and less than two fundamental flavors, nonperturbative effects lead to a brane configuration that is topologically a torus. Using the description of the orientifold six-planes as D_n singularities we discuss the Higgs branches for $\mathcal{N} = 2$ brane configurations with Sp/SO gauge groups and the related $\mathcal{N} = 1$ theories with tensor representations.

Keywords: D-branes, Brane Dynamics and Gauge Theories.

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Contents

1. Introduction

The study of supersymmetric gauge theories in various dimensions realized by brane configurations has been a very active research area in recent times. Many new results concerning the non-perturbative behavior of gauge theories have been obtained. A recent review of the techniques involved and a summary of relevant references was given in [1].

To study four-dimensional supersymmetric gauge theories one commonly uses the following building blocks. In type IIA string theory one considers a collection of parallel fivebranes and fourbranes stretched between them. The world volume of the fourbranes is then bounded by the fivebranes and thus of finite extent in one direction (commonly labeled x^6). At sufficiently low energies the physics of the fourbranes is then described by a four-dimensional gauge theory. Such a configuration will leave eight supercharges unbroken giving rise to a four-dimensional $\mathcal{N} = 2$ gauge theory [2]. If one rotates the fivebranes such that the rotation lies in an $SU(2)$ subgroup of the $SO(4)$ rotation group transverse to the fivebranes, one breaks another half of the supersymmetries [3]. In this way one can construct brane configurations corresponding to four-dimensional $\mathcal{N} = 1$ supersymmetric gauge theories. Sixbranes in between two fivebranes with an orientation that preserves the supersymmetries give rise to matter transforming in the fundamental representation of the gauge group. Upon going from type IIA string theory to M-theory the fourbranes become themselves fivebranes that are wound around the eleventh dimension. The whole brane configuration becomes a single fivebrane with worldvolume $R^4 \times \Sigma$ with Σ being a Riemann surface [4]. Information about the nonperturbative behavior of the gauge theory is encoded in this Riemann surface.

An important ingredient in these brane constructions are orientifold planes. They can be introduced as four-orientifolds parallel to the fourbranes or as six-orientifolds parallel to the sixbranes. Both possibilities are compatible with supersymmetry and give rise to orthogonal or symplectic gauge groups. Six-orientifolds are of particular interest. It has been shown that by placing a six-orientifold on top of a fivebrane one obtains gauge theories with $SU(N_c)$ gauge group and matter transforming in the symmetric or antisymmetric representation. The fivebrane can also divide the six-orientifold in two and this gives rise to a chiral $\mathcal{N} = 1$ theory [5].

This paper is devoted to the investigation of several aspects of brane configurations with six-orientifolds. In section 2 we use a configuration consisting of three parallel fivebranes and a six-orientifold on top of the central one. We also include a number of sixbranes. We obtain $\mathcal{N} = 2$ $SU(N_c)$ gauge theories with one flavor of symmetric or antisymmetric matter and N_f fundamentals. From the brane configurations we derive the curves parameterizing the Coulomb branch of these theories. We note that there is the interesting effect of non-perturbative mass generation for $N_f = N_c - 3$ in the case with a symmetric flavor and for $N_f = N_c + 1$ in the case with an antisymmetric flavor.[1]

In section 3 we rotate the outer fivebranes and break to $\mathcal{N} = 1$. The corresponding brane configuration in M-theory is parameterized by a $\mathcal{P}^1$. However, it turns out that in the case with an antisymmetric flavor and one or zero fundamental flavors the brane configuration is not birational to a sphere. We go on to investigate these cases further in section 4. There we argue that non-perturbative effects due to the orientifold generate an additional handle and that the brane configuration is actually a genus one curve. We show that the asymptotic behavior is consistent with the assumption of a genus one curve. In section 5 we briefly comment on the chiral brane configuration. Section 6 discusses the Higgs branches of $\mathcal{N} = 2$ brane configurations corresponding to orthogonal and symplectic gauge theories. We use there the description of six-orientifolds as D_n singularities [7]. It was suggested in [8] that the six-orientifold giving rise to orthogonal gauge groups can be described by an D_{N_f+4} singularity. It is important that in this case the singularity can only be resolved down to D_4. We compute the dimension of

[1] A similar effect was found in [6].

the Higgs branch using this description and show that it indeed coincides with field theory. In section 7 we discuss the Higgs branches of $\mathcal{N}=1$ SO/Sp gauge theories with tensor representations. Again using the description of the six-orientifold as a D_n singularity we can compute the dimensions of the various Higgs branches.

After this work was completed we learned of independent work [9] that partially overlaps with some results in sections 3, 5 and 6.

2. Curves for $\mathcal{N}=2$ $SU(N_c)$ with a tensor flavor and N_f fundamentals

2.1. Symmetric flavor

In this section, the Seiberg-Witten curve for an $\mathcal{N}=2$ $SU(N_c)$ gauge theory with a flavor of symmetric and N_f fundamental flavors is constructed. Following [2], we will derive it by lifting a certain type IIA brane configuration to M-theory. The basic brane configuration is that considered in [10]: three NS fivebranes (012345) and N_c fourbranes suspended between them (01236), in the presence of an orientifold sixplane of +4 Ramond charge (0123789). We have indicated in brackets the type IIA directions in which each object extends. There will be in addition $2N_f$ sixbranes parallel to the orientifold (see fig. 1). In order to describe the lifting to M-theory of this configuration, it is common to introduce complex coordinates $v = x_4 + ix_4$ and $s = (x_6 + ix_{10})/R$

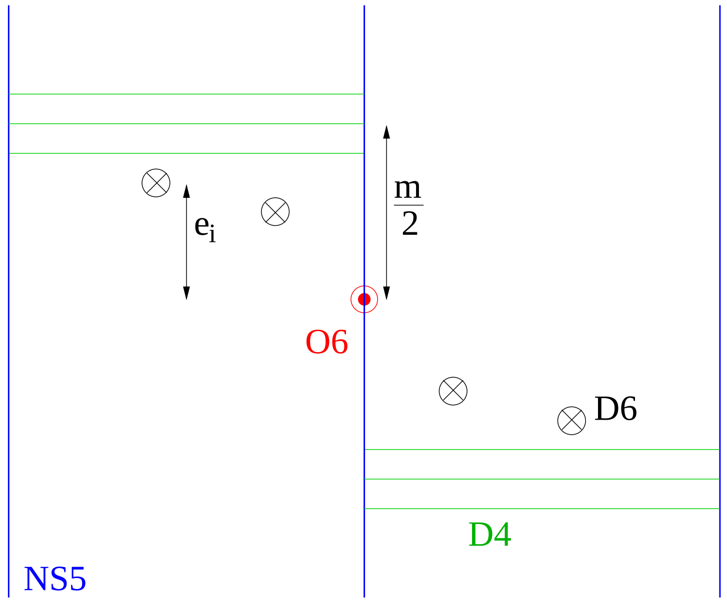

Figure 1: An $\mathcal{N}=2$ brane configuration with three fivebranes. On top of the middle fivebrane there is a six-orientifold. Such configurations give rise to $SU(N_c)$ gauge theories with matter hypermultiplets either in the symmetric or antisymmetric representation depending on the sixbrane charge of the orientifold. In addition there are some sixbranes giving rise to hypermultiplets in the fundamental representation.

where x_{10} denotes the eleventh dimension of M-theory and R is its radius. A orientifold sixplane of positive Ramond charge and $2N_f$ sixbranes can be described in M-theory by [10]

$$xy = (-1)^{N_f} v^4 \prod_{k=1}^{N_f} (v^2 - e_k^2), \tag{2.1}$$

where v^4 comes from the orientifold six-plane, and x, y are complex coordinates such that for small or fixed x, $y \sim e^{-s}$, and for small or fixed y, $x \sim e^s$. The parameters e_k correspond to the position of the sixbranes in the v-plane [2]. The orientifold action in these coordinates is $(x, y, v) \to (y, x, -v)$. Finally, it is convenient to define

$$\begin{aligned} j_1(v) &= \prod_{k=1}^{N_f} (v - e_k), \\ j_2(v) &= (-1)^{N_f} \prod_{k=1}^{N_f} (v + e_k). \end{aligned} \tag{2.2}$$

The M-theory curve describing a brane configuration with several fivebranes in a background space of the form (2.1) was given in [2]. We just have to impose in addition invariance under the orientifold projection. We obtain the following curve for the configuration with three fivebranes

$$y^3 + y^2 p(v) + y v^2 j_1(v) q(v) + v^6 j_1^2(v) j_2(v) = 0, \tag{2.3}$$

where $q(v) = p(-v)$ and $p(v) = \prod_{i=1}^{N_c} (v - a_i) = v^{N_c} + \frac{1}{2} N_c m v^{N_c - 1} + u_2 v^{N_c - 2} + ...$ The parameters a_i represent the positions of the fourbranes on the v-plane. Combinations of them give m, the mass of the symmetric flavor, and u_k, $k = 2, \cdots, N_c$, the $SU(N_c)$ casimirs. The sixbranes induce N_f fundamental hypermultiplets for the $SU(N_c)$ gauge theory. Notice that the masses of these hypermultiplets are given by (see fig. 1)

$$m_k = \frac{m}{2} - e_k. \tag{2.4}$$

The $SU(N_c)$ theory with a symmetric and N_f fundamental flavors has baryonic operators $B_n = X^n Q^{N_c - n} Q^{N_c - n}$, $\tilde{B}_n = \tilde{X}^n \tilde{Q}^{N_c - n} \tilde{Q}^{N_c - n}$, where $X, \tilde{X}$ represent the fields in the symmetric representation and $Q, \tilde{Q}$ the quarks. When B_n or $\tilde{B}_n$ get an expectation value the initial $SU(N_c)$ theory breaks to $SO(n)$ with $N_f - (N_c - n)$ flavors. In the brane language, the baryonic branches will correspond to factorizing the central fivebrane. The curve (2.3) factorizes into

$$(y + v^2 j_1)(y^2 + y(p - v^2 j_1) + v^4 j_1 j_2) = 0 \ , \tag{2.5}$$

when

$$p(v) - v^2 j_1(v) = q(v) - v^2 j_2(v) \ , \tag{2.6}$$

which implies that $p - v^2 j_1$ only contains even powers of v. The second factor in (2.5) corresponds to the curve of an $SO(N_c)$ $\mathcal{N} = 2$ gauge theory with N_f flavors [11]. This is the expected breaking associated with expectation values for B_{N_c} and $\tilde{B}_{N_c}$.

The first term in (2.5) represents the factorized central fivebrane. From field theory we get that the quarternionic dimension of this baryonic branch is one. In the brane language three of the four real parameters can be related to the x_7, x_8, x_9 position of the fivebrane. The field theory has a global $U(1)_X$ symmetry acting only on the field in the two-index tensor representation. The fourth parameter can be understood as the Goldstone mode for this $U(1)_X$ symmetry. However, this $U(1)_X$ is not realized geometrically in the brane configuration. Therefore we can not see the fourth real parameter as a geometric quantity. It should be related to the integration of the chiral antisymmetric two-tensor field along the world-volume, when suitably regularized.

For N_c odd the curve (2.3) factorizes as

$$(y - v^2 j_1)(y^2 + y(p + v^2 j_1) - v^4 j_1 j_2) = 0 \ , \tag{2.7}$$

when

$$p(v) + v^2 j_1(v) = -(q(v) + v^2 j_2(v)) \ , \tag{2.8}$$

which in this case implies $p + v^2 j_1 = vB(v^2)$. The second factor in (2.7) describes a brane configuration with two fivebranes and an odd number of fourbranes in the space (2.1). By redefining $y \to vy$ we obtain the Seiberg-Witten curve associated with an odd orthogonal theory with N_f flavors in its standard form [11].[2]

Notice that the conditions for factorization (2.5) and (2.7) are not simply $q = p$ and $q = -p$ for N_c even and odd respectively, as one would have naively expected. In particular for $N_f = N_c - 3$ (2.5) and (2.7) imply that

$$m \sim \Lambda_{\mathcal{N}=2}, \tag{2.9}$$

where we have restored the dependence on the dynamical scale of the theory, which we had set to 1 in the above. An analogous shift in the mass of a bifundamental flavor was encountered when analyzing the Seiberg-Witten curves for an $SU(N_c) \times SU(N_c)$ gauge theory [6]. As in [6] we interpret this as the non-perturbative generation of a mass for the symmetric flavor, which has to be canceled for the factorization to occur. It also means that the location of the root of the baryonic branch where the middle fivebrane detaches suffers a non-perturbative correction. A somewhat analogous shift of a Higgs branch root will appear for the $N_f/2$-th branch of $\mathcal{N} = 2$ Sp theories in section 6. It was also noted already in [12]. Notice that (2.4) suggests that also the quarks receive non-perturbative corrections to their mass.

The other baryonic branches appear when in addition to the factorization (2.5) or (2.7), we also factorize $p(v) \pm v^2 j_1(v)$ as $v^{N_c - n}\tilde{p}(v)$. This amounts to putting the $SO(N_c)$ gauge theory at the origin of its $(N_c - n)$-Higgs branch. We may then blow up the D_{N_f+4} singularity as described in section 6, taking us onto the Higgs branch of

[2]The redefinition $y \to vy$ does not have meaning in the M-theory context. It would be equivalent to resolve part of the D_4 singularity associated with the orientifold. This is however not allowed [8], [10].

the $SO(N_c)$ theory. In this way we obtain exactly the number of moduli necessary to describe the B_n and $\tilde{B}_n$ baryonic branches of the original $SU(N_c)$ theory, since their quaternionic dimension equals the dimension of the $(N_c - n)$-Higgs branch of an SO theory with N_f flavors, plus one. The additional one corresponds to the factorized central fivebrane as before.

2.2. Antisymmetric flavor

We consider now the same brane configuration as in the previous section, but in the background of an orientifold sixplane of -4 Ramond charge. As explained in [10] this configuration induces an $\mathcal{N} = 2$ $SU(N_c)$ gauge theory with an antisymmetric flavor on the world-volume of the fourbranes. We include again the presence of $2N_f$ sixbranes, which will provide N_f fundamental flavors for the $SU(N_c)$ theory. An orientifold sixplane of negative charge and $2N_f$ coincident sixbranes are described in M-theory by compactification in a complex two-dimensional space with a D_{N_f} singularity [7]. The sixbranes can be taken in pairs away from the orientifold. As a complex manifold, this is represented by a deformed D_{N_f} surface [13]

$$a^2 + b^2 z = \frac{4}{z} \left(\prod_{k=1}^{N_f} (z + e_k^2) - \prod_{k=1}^{N_f} e_k^2 \right) - 4b \prod_{k=1}^{N_f} e_k \ . \tag{2.10}$$

The parameters e_k describe the position of the sixbranes in the z direction.

As in [10], instead of (2.10) we will use a space that is birationally equivalent to it and which only provides a complete description of (2.10) far from the orientifold

$$xy = (-1)^{N_f} v^{-4} \prod_{k=1}^{N_f} (v^2 - e_k^2) \ . \tag{2.11}$$

In this space we impose invariance under the orientifold projection $(y, x, v) \to (x, y, -v)$. The different spaces are related by $a = v(y - x)$, $b = x + y + 2v^{-2} \prod_{k=1}^{N_f} e_k$, $z = -v^2$, which corresponds to choosing $\mathbb{Z}_2$ invariant coordinates in (2.11).

In the auxiliary space (2.11), the most general Riemann surface associated with a configuration of three fivebranes is

$$y^3 + y^2(p + Bv^{-1} + 3Av^{-2}) + yv^{-2} j_1(q - Bv^{-1} + 3Av^{-2}) + v^{-6} j_1^2 j_2 = 0 \ , \tag{2.12}$$

where again $q(v) = p(-v)$ and $p(v) = \prod_{i=1}^{N_c} (v - a_i)$, with N_c the number of fourbranes suspended between the fivebranes. The polynomials $j_1(v), j_2(v)$ are defined as in (2.2). The terms Bv^{-1}, $3Av^{-2}$ are not related with fourbranes positions. They are allowed by the presence of negative powers of v in (2.11). However the fact that they can not be fixed with the information contained in (2.11), is a sign that this space does not provide a good description of the orientifold background close to the origin. We use the space (2.11) as a tool for obtaining a very restricted ansatz for the desired curve [10]. Once we have the ansatz, the extra coefficients are determined by imposing that the curve can be written as a polynomial in the standard D_{N_f} surface (2.10), where no negative powers of v appear. The result is

$$B = \Lambda_{\mathcal{N}=2}^{N_c+2-N_f} \sum_{k=1}^{N_f} \prod_{l\neq k} -e_l \ , \qquad A = \Lambda_{\mathcal{N}=2}^{N_c+2-N_f} \prod_{k=1}^{N_f} -e_k \ . \tag{2.13}$$

where we have restored the dependence on the dynamical scale $\Lambda_{\mathcal{N}=2}$ of the $SU(N_c)$ theory. Both coefficients are proportional to the dynamical scale, indicating their origin in the strong coupling dynamics close to the orientifold. The parameters e_k are again related to quark masses by (2.4).

As a check of the proposed curves we will analyze deformations of the previous curve associated with baryonic branches of the $\mathcal{N}=2$ $SU(N_c)$ theory with an antisymmetric flavor and N_f quark flavors. This theory contains baryons of the form $B_n = X^n Q^{N_c-2n}$ and $\tilde{B}_n = \tilde{X}^n \tilde{Q}^{N_c-2n}$. When N_c is even the baryons $B_{N_c} = X^{N_c/2}$, $\tilde{B}_{N_c} = \tilde{X}^{N_c/2}$, break the initial theory down to an $Sp(N_c)$ theory with N_f hypers in the fundamental representation. In agreement with this, (2.12) factorizes as

$$(y + v^{-2}j_1)(y^2 + y(p - \tilde{j}_1 + 2Av^{-2}) + v^{-4}j_1 j_2) = 0 \ , \tag{2.14}$$

when

$$p(v) - \tilde{j}_1(v) = q(v) - \tilde{j}_2(v) \ . \tag{2.15}$$

We have defined $\tilde{j}_2(v) = \tilde{j}_1(-v)$ and $\tilde{j}_1(v) = v^{-2}(j_1(v) - Bv - A)$. The factorization condition insures that only even powers of v appear in the second factor of (2.14), and we obtain the curve for an $Sp(N_c)$ $\mathcal{N}=2$ theory with N_f flavors [11] as expected. This factorization property can alternatively be used to fix the coefficients A and B.

When N_c is odd the highest baryon operators are $B_{N_c-1} = X^{(N_c-1)/2}Q$, $\tilde{B}_{N_c-1} = \tilde{X}^{(N_c-1)/2}\tilde{Q}$, which break the $SU(N_c)$ theory to $Sp(N_c{-}1)$ with $N_f{-}1$ flavors. According to this, for $e_k \neq 0$ and N_c odd, the curve (2.12) does not factorize for any value of the casimirs. However when at least one $e_k = 0$, we have $A = 0$ and the curve factorizes as

$$(y - v^{-2}j_1)(y^2 + y(p + \tilde{j}_1 + 2Bv^{-1}) - v^{-4}j_1 j_2) = 0 \ , \tag{2.16}$$

provided that

$$p(v) + \tilde{j}_1(v) = -(q(v) + \tilde{j}_2(v)) \ . \tag{2.17}$$

The conditions (2.15), (2.17) for $N_f < N_c + 1$ imply that the antisymmetric flavor must be massless for the factorization to occur. For $N_f = N_c + 1$ we have again a non-perturbative shift in the mass of the antisymmetric (see (2.9)). Thus when (2.17) is fulfilled, $e_k = 0$ implies a massless fundamental if $N_f < N_c + 1$ and $m_k \sim \Lambda_{\mathcal{N}=2}$ when $N_f = N_c + 1$.

From condition (2.16) we get $p(v) + \tilde{j}_1(v) = vb(v^2)$. Since one of the e_k must be zero we have $j_1(v) = vj_1'(v)$, where j_1' is the polynomial associated with a background space of $N_f - 1$ sixbranes. Redefining then $y \to vy$, the second factor in (2.16) reproduces the Seiberg-Witten curve for $Sp(N_c - 1)$ theory with $N_f - 1$ flavors. In this case we

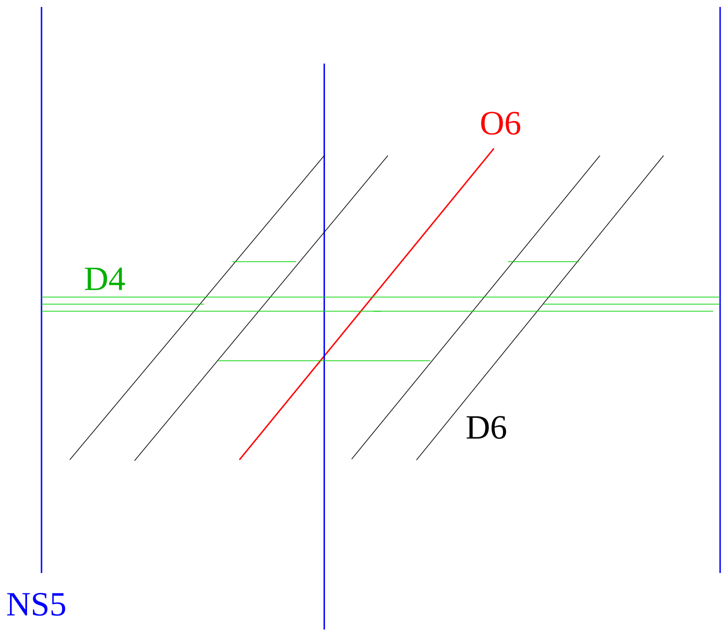

Figure 2: Baryonic branch in the case N_c odd.

interpret the first factor in (2.16) as corresponding to the factorized central fivebrane with a fourbrane attached to it. One of the fourbranes has to remain attached to the middle fivebrane otherwise we would have an odd number of fourbranes intersecting an orientifold that projects onto symplectic groups, which is not consistent. The quaternionic dimension of the B_{N_c-1}, $\tilde{B}_{N_c-1}$ baryonic branches is N_f. This is indeed correctly reproduced by the brane configuration. One modulus corresponds to the position of the central fivebrane together with the attached fourbrane and $N_f - 1$ to the possible $\mathbb{Z}_2$-symmetric breakings of this attached fourbrane in the $2N_f$ sixbranes (see fig. 2).

3. Curves for $\mathcal{N} = 1$ $SU(N_c)$ with a tensor flavor and N_f fundamentals

Now we want to obtain the curves for the theories with $\mathcal{N} = 1$ supersymmetry. A way to achieve this is to introduce a mass for the chiral multiplet in the adjoint representation of the $\mathcal{N} = 2$ theory. As is well-known, in the the brane configuration this corresponds to rotating fivebranes from their original orientation along the $4, 5$ directions towards the $8, 9$ plane. We introduce a new complex variable $w = x^8 + ix^9$. In our case we have three fivebranes, of which the outer two can be rotated in a $\mathbb{Z}_2$ symmetric manner. Thus the left fivebrane will be described asymptotically for large values of v by $v/w = \mu$ and the right one by $v/w = -\mu$. Furthermore we are interested in the case when the curve can be parameterized rationally by a $\mathcal{P}^1$ whose coordinate we denote by λ. We will furthermore restrict ourselves to the case when all additional sixbranes lie at $v = 0$ for the moment.

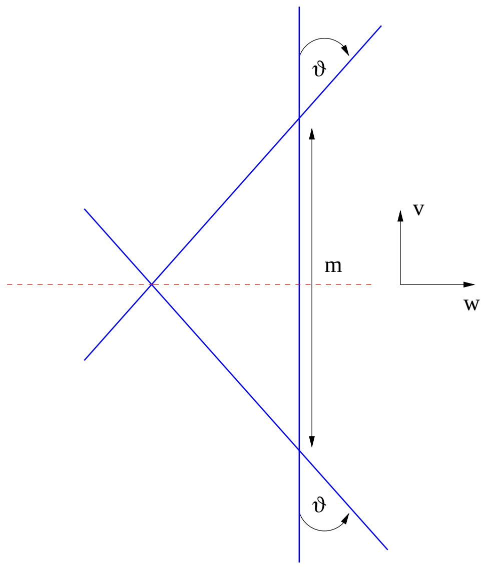

Figure 3: Projection onto the (v, w)-plane of the rotated configuration.

The tree level superpotential associated with the rotated brane configuration can be obtained from the $\mathcal{N} = 2$ superpotential by integrating out the massive adjoint multiplet. The result is

$$W_{tree} = -\frac{1}{2\mu}\left((X\tilde{X})^2 + Q\tilde{X}X\tilde{Q} + (Q\tilde{Q})^2\right) + mX\tilde{X} + \frac{m}{2}Q\tilde{Q}, \tag{3.1}$$

with m as depicted in fig. 3. The parameter μ is the only one carrying $U(1)_{89}$-charge associated with rotations in the 89 plane. The $\mathcal{N} = 1$ curve can be thought of as a deformation of the $\mathcal{N} = 2$ curve. However, because of the charge μ is carrying, it can not appear in the projection of the $\mathcal{N} = 1$ curve onto the the (y, v)-plane [14]. It follows that this projection has the same form as the $\mathcal{N} = 2$ curve. Consider now the curves for the theories with a symmetric flavor and N_f fundamentals and the theories with an antisymmetric flavor and $N_f + 2$ flavors. From the expressions we derived in the previous section one sees that, by appropriate rescalings of y, these curves can be brought into the form

$$v^M y^3 + y^2 p(v) + yp(-v) + (-1)^M v^M = 0\,. \tag{3.2}$$

This allows us to treat the cases for the antisymmetric and the symmetric tensor at once. The orientifold is now assumed to act as $(y, v) \to (1/y, -v)$. We define $M = N_f + 2$ in case of the symmetric and $M = N_f - 2$ in case of the antisymmetric representation. In these coordinates one sees M semi-infinite fourbranes to the left and to the right of the brane configuration located at $v = 0$. It is important to note that in the case of the symmetric flavor two of these fourbranes represent the effects of the orientifold rather than matter in the fundamental representation [15]. The cases $N_f = 0, 1$ and an antisymmetric flavor are special. We postpone their study until further in this section.

Now we want to investigate the rotated brane configurations [16]. In terms of λ we can assume

$$
\begin{aligned}
v &= \frac{b}{\lambda-1} + \frac{\tilde{b}}{\lambda} + \frac{b}{\lambda+1}\,, \\
w &= \frac{b\,\mu}{\lambda-1} - \frac{b\,\mu}{\lambda+1} + 2b\,\mu\,, \\
y &= A\left(\frac{\lambda-1}{\lambda+1}\right)^{N_c-M} .
\end{aligned}
\tag{3.3}
$$

The form of v follows since we have three fivebranes and correspondingly we demand three poles of order one for v as a function of λ. Two of the fivebranes are interchanged by the action of the orientifold. On the sphere this orientifold acts by $\lambda \to -\lambda$. Since the middle fivebrane does not have a mirror image, it must approach asymptotically one of the two fixed points $v=\infty,\ y=\pm 1$. The middle fivebrane must therefore be represented by a pole at $\lambda = 0$ (equivalently we could choose $\lambda=\infty$). Without loss of generality, the positions of the poles for the outer fivebranes are set to $\lambda=\pm 1$. The coefficients of these poles are forced to be equal by the orientifold $\mathbb{Z}_2$.

The form of w is given by similar considerations and by the asymptotic conditions $w=\pm\mu v$ at $\lambda=\pm 1$. We also included a shift in w such that on the middle fivebrane w approaches zero for large values of v. Using the form (3.2) it follows that y has a zero of order N_c-M at $\lambda=1$, a pole of oder N_c-M at $\lambda=-1$ and goes to a constant for $\lambda\to 0$.

The constants $A, b, \tilde{b}$ can be fixed by inserting (3.3) into (3.2) and examining the asymptotics at $\lambda=1$ and $\lambda=0$ (the asymptotic expansion around $\lambda=-1$ is equivalent to the one at $\lambda=1$). Around $\lambda=1$ we have to leading and first subleading order

$$
\begin{aligned}
&\frac{1}{(\lambda-1)^M}\Big((-1)^{N_c}\frac{Ab^{N_c}}{2^{N_c-M}} + (-1)^{N_c} b^M\Big) + \\
&\quad + \frac{1}{(\lambda-1)^{M-1}}\Big((-1)^{N_c}\frac{Ab^{N_c-1}}{2^{N_c-M+1}}(bM + 2\tilde{b}N_c - N_c m) + \\
&\quad + (-1)^M M b^{M-1}(\frac{b}{2}+\tilde{b})\Big) + A^2 b^{N_c}(\lambda-1)^{N_c-2M} + \cdots = 0\,.
\end{aligned}
\tag{3.4}
$$

For a theory with asymptotic freedom we must have $N_c > M$, which has been assumed in (3.4). The last term contributes to the subleading order for $N_c = M+1$.

Around $\lambda=0$ we have

$$
\begin{aligned}
&\frac{1}{\lambda^{N_c}}\Big((-1)^{2N_c-2M}A^2\tilde{b}^{N_c} + (-1)^{2N_c-M}A\tilde{b}^{N_c}\Big) + \\
&\quad + \frac{1}{\lambda^{N_c-1}}\Big(4(-1)^{2N_c-2M}A^2\tilde{b}^{N_c}(M-N_c) + \\
&\quad + \frac{N_c m}{2}(-1)^{2N_c-2M}A^2\tilde{b}^{N_c-1} + 2(-1)^{2N_c-M}A\tilde{b}^{N_c}(M-N_c) - \\
&\quad - \frac{N_c m}{2}(-1)^{2N_c-M}A\tilde{b}^{N_c-1}\Big) +
\end{aligned}
$$

$$+\frac{1}{\lambda^M}\left((-1)^{3N_c-3M}A^3\tilde{b}^M+(-1)^M\tilde{b}^M\right)+\cdots=0\,. \tag{3.5}$$

Again, the last term contributes to the subleading order only for $N_c = M+1$. The terms at lower orders do not give new conditions for the constants in (3.3) but determine the casimirs u_i. These equations are solved by

$$\begin{aligned}
A &= (-1)^{M+1}\Lambda_{\mathcal{N}=2}^{N_c-M}\,,\\
b^{N_c-M} &= (-1)^{N_c}(2\Lambda_{\mathcal{N}=2})^{N_c-M}\,,\\
\tilde{b} &= \frac{N_c m}{2(N_c-M)} \qquad \text{if } \ N_c > M+1\,,\\
\tilde{b} &= (-1)^M\Lambda_{\mathcal{N}=2}+\frac{N_c m}{2} \quad \text{if } \ N_c = M+1\,.
\end{aligned} \tag{3.6}$$

Again we restored the dependence on the scale $\Lambda_{\mathcal{N}=2}$. It is interesting to note that the coefficient for the middle fivebrane is proportional to the mass m. This also means that for zero mass the pole of v corresponding to the middle fivebrane is absent, which implies the curve factorizes. This is in line with field theory expectations – the baryonic branches will open up when the mass of the tensor vanishes. Therefore for zero mass with the central fivebrane factorized the curves are the same as the corresponding curves for SO or Sp theories with tensor matter [17, 18].

For $N_c = M+1$ however, the coefficient for the middle fivebrane vanishes when $m = (-1)^{M+1}2\Lambda_{\mathcal{N}=2}/N_c$. This is of course the same non-perturbative shift in the mass as we noted in the previous section.

Let us analyze the behavior of the solutions at $v = 0$. In terms of λ this corresponds to the three points $\lambda = \infty$ and $\lambda = \pm\sqrt{\tilde{b}/(2b+\tilde{b})}$. For generic $\tilde{b}$ we therefore get three different values $y = (A, y_+, y_-)$. This means that the projection of the curve to the (y, v) plane at $v = 0$ is given by $(y-A)(y-y_+)(y-y_-) = 0$. This is possible only if the polynomial $p(v)$ factorizes as $p(v) = v^M\tilde{p}(v)$. In the coordinates we are using we have M semi-infinite fourbranes to the left and to the right of the brane configuration. The particular form of $p(v)$ means that we first have to bring M fourbranes to $v = 0$ and reconnect them with the semi-infinite ones. Only then we can perform the rotation to $\mathcal{N} = 1$.

We will study now the cases with $N_f = 0, 1$ and an antisymmetric flavor. Instead of (3.2), the (y, v) projection of the curve for $N_f = 1$ is given by

$$y^3+y^2(vp(v)+1)+y(vp(-v)-1)-1=0\,, \tag{3.7}$$

and for $N_f = 0$ by

$$y^3+y^2(v^2p(v)+3)+y(v^2p(-v)+3)+1=0\,. \tag{3.8}$$

We consider first $N_f = 1$. The asymptotic conditions can be taken over from (3.4) and (3.5). This would lead to a similar curve as found above. However, now we have to take into account that the curve takes a definite form at $v = 0$ and we can not adjust some order parameter to solve the equation. Indeed for $v = 0$ we need to satisfy $(y-1)(y+1)^2 = 0$. The zeroes of v are given as above. For $\lambda = \infty$, $y = -1$ since from the asymptotic conditions $A = -1$. For generic $\tilde{b}$ the other two solutions of $v = 0$ will not give $y = \pm 1$. Moreover, we already have one solution at $y = -1$. We would need to have two more solutions $\lambda_\pm$ with $y = \pm 1$. The orientifold action exchanges λ_+ with λ_- but leaves the y-values ± 1 fixed. Thus if $y(\lambda_+) = 1$ then also $y(\lambda_-) = 1$. This argument shows that we can not achieve the required structure of the solutions for y through the parameterization by a sphere. We will argue in the next section that the curves in this case are topologically a torus rather than a sphere. When the mass of the antisymmetric vanishes the middle fivebrane can again factorize as described in the previous section. Up to the detached middle fivebrane the curve can be rotated to $\mathcal{N} = 1$ in the same way as the curves for $Sp(N_c)$ when N_c is even or for $Sp(N_c - 1)$ when N_c is odd.

Consider now the case with one antisymmetric flavor and no fundamentals at all. Again we have the asymptotic conditions (3.4) and (3.5), implying $A = -1$, $b^{N_c+2} = (-2)^{N_c+2}$ and $\tilde{b} = m/2$. Also here we have to investigate carefully the behavior at $v = 0$. This time the (y, v) projection give rise to the triple point $(y+1)^3 = 0$. Now all the zeroes of v in λ have to give $y = -1$. Thus we have the equation

$$\left(\frac{\lambda - 1}{\lambda + 1}\right)^{N_c+2} = 1\,, \tag{3.9}$$

with the solutions

$$\lambda = i \cot\left(\frac{n\pi}{N_c + 2}\right)\,, \tag{3.10}$$

where $n = 0, \cdots, N_c+1$. $n = 0$ corresponds to $\lambda = \infty$, the other two solutions $\lambda_\pm$ come as the pair $n, N_c + 2 - n$ for fixed n. Note that this also implies specific values for the mass m of the antisymmetric. In the next section we will argue that for generic m the curve is again topologically equivalent to a torus. For N_c even and $n = N_c/2 + 1$ we get $\lambda_\pm = 0$. This implies $\tilde{b} = m = 0$. This is consistent with the fact that only for even N_c the middle fivebrane can factorize. For this case the other component of the curve will be equivalent to that of the Sp gauge theory that appears on the baryonic branch.

Finally, let us consider the form of the curve with an (anti)symmetric flavor and with nontrivial masses for the fundamental flavors. We switch back to a description where the ambient space is given by (2.11). For simplicity, we will set the mass of the tensor to zero, which corresponds to working at a point where the $\mathcal{N} = 2$ curve factorizes.[3] We will also take N_c to be even, thus the resulting curves will also describe $SO(2n_c)$ and $Sp(2n_c)$ gauge theories where $N_c = 2n_c$.

With an antisymmetric flavor the curve takes the form

$$v = \frac{b}{\lambda - 1} + \frac{b}{\lambda + 1}$$

[3]For $N_c = M + 1$ and M defined as before, we should fix the mass of the (anti)symmetric flavor to the quantum corrected value $m = (-1)^{N_f+1}\frac{2\Lambda_{\mathcal{N}=2}}{N_c}$.

$$
\begin{aligned}
w &= \frac{\mu b}{\lambda - 1} - \frac{\mu b}{\lambda + 1} \\
y &= A v^{-2} \prod_{i=1}^{N_f} \frac{\lambda^2 + 1 - \lambda c_i}{\lambda^2 - 1} \left(\frac{\lambda - 1}{\lambda + 1} \right)^{2n_c + 2 - N_f} ,
\end{aligned}
\tag{3.11}
$$

where we have defined

$$
\begin{aligned}
c_i^2 &= \frac{4(b^2 + m_i^2)}{m_i^2} \\
b^{4n_c+4} &= (2\Lambda_{\mathcal{N}=2})^{4n_c+4-2N_f} \prod_{i=1}^{N_f} m_i^2 (1 + c_i/2)^2 \\
A^2 &= \Lambda_{\mathcal{N}=2}^{4n_c+4-2N_f} \prod_{i=1}^{N_f} m_i^2 ,
\end{aligned}
\tag{3.12}
$$

with m_i the masses of the fundamental flavors. The form of (3.11) is determined by asking that $x + y = p(v^2)$.

For the symmetric case, the curve is

$$
\begin{aligned}
v &= \frac{b}{\lambda - 1} + \frac{b}{\lambda + 1} \\
w &= \frac{\mu b}{\lambda - 1} - \frac{\mu b}{\lambda + 1} \\
y &= A v^2 \prod_{i=1}^{N_f} \frac{\lambda^2 + 1 - \lambda c_i}{\lambda^2 - 1} \left(\frac{\lambda - 1}{\lambda + 1} \right)^{2n_c - 2 - N_f} ,
\end{aligned}
\tag{3.13}
$$

where we have defined

$$
\begin{aligned}
c_i^2 &= \frac{4(b^2 + m_i^2)}{m_i^2} \\
b^{4n_c-4} &= (2\Lambda_{\mathcal{N}=2})^{4n_c-4-2N_f} \prod_{i=1}^{N_f} m_i^2 (1 + c_i/2)^2 \\
A^2 &= \Lambda_{\mathcal{N}=2}^{4n_c-4-2N_f} \prod_{i=1}^{N_f} m_i^2 .
\end{aligned}
\tag{3.14}
$$

After a change of variables, these curves coincide with those found in [17, 18] where the related SO and Sp gauge theories were studied.

4. $\mathcal{N} = 1$ $SU(N_c)$ with an antisymmetric and $N_f = 0, 1$ fundamental flavors

In the last section we have seen that the genus zero ansatz for the rotated curve associated with $\mathcal{N} = 1$ $SU(N_c)$ with a massive antisymmetric flavor and $N_f = 0, 1$ fails for generic values of the antisymmetric mass. The rotated brane configuration for

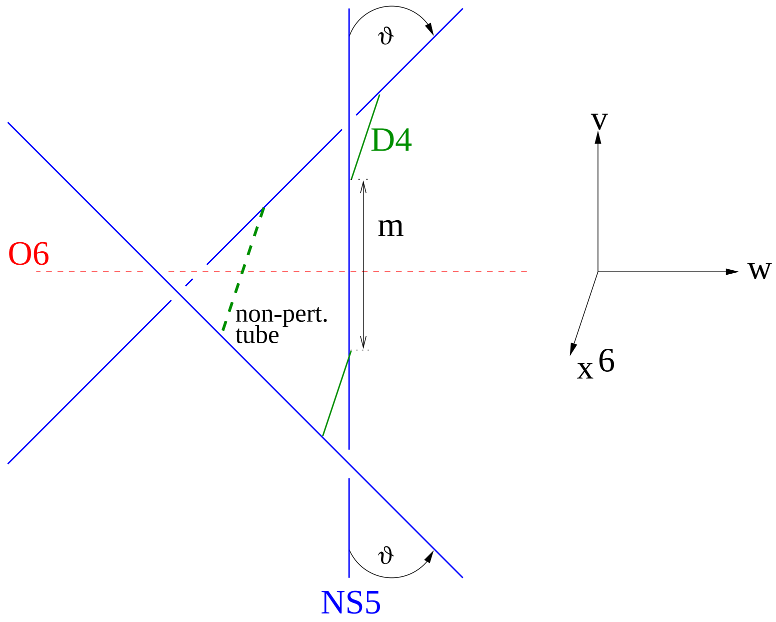

Figure 4: This figure illustrates what happens when one tries to rotate the brane configuration for $SU(N_c)$ gauge theory with a massive flavor in the antisymmetric representation and no fundamental flavors.

$m \neq 0$ is shown in fig. 4. It is important to analyze it in a neighborhood of the point $(v = 0, w = \mu m/2)$. Around this point the configuration reduces to two fivebranes rotated symmetrically with respect to the orientifold sixplane. We recall now a crucial feature of the curves associated to $\mathcal{N} = 2$ theories with symplectic gauge group. For $N_f = 0, 1$ they possess one more handle than expected from the number of fourbranes present [19], [10]. The additional handle originates from non-perturbative effects due to the orientifold plane and does not have a physical $U(1)$ associated with it. This feature appears both when the symplectic projection is imposed by an orientifold fourplane of positive Ramond charge, or an orientifold sixplane of negative Ramond charge. Since this effect only depends on the orientifold plane, it must also be present for a configuration of two fivebranes and no fourbranes. We will assume that even when the fivebranes are rotated, a non-perturbative tube is generated connecting them. This suggests that the $\mathcal{N} = 1$ brane configuration of fig. 4 will define a genus one Riemann surface instead of a genus zero surface. The extra handle will come from the non-perturbative spike created by the orientifold sixplane of negative Ramond charge around $w = \mu m/2$.

We will analyze first the case $N_f = 0$. Let us consider the asymptotic behavior that the rotated curve should have at $v \to \infty$ and $v = 0$. It will be convenient to use the curves in the form derived in section 2, i.e. without rescaling the coordinate y as we did in the previous section. The behavior as $v \to \infty$ should be

$$\begin{aligned} &i) \quad y \to -v^{N_c}, && w \to \mu v, \\ &ii) \quad y \to (-1)^{N_c+1} v^{-2}, && w \to 0, \\ &iii) \quad y \to (-1)^{N_c+1} v^{-N_c-4}, && w \sim -\mu v \;. \end{aligned} \tag{4.1}$$

where i), ii) and iii) correspond to the left, central and right fivebranes respectively. The behavior in a neighborhood of $v = 0$ is fixed by the non-perturbative effects associated with the orientifold to be

$$y \sim -v^{-2} \ . \tag{4.2}$$

In the following we want to show that a genus one curve can satisfy (4.1), (4.2); conditions that a genus zero curve was unable to meet for generic $m \neq 0$. We will comment at the end about how to relate the elliptic modulus τ of the genus one curve with the gauge theory parameters. We know however that the specific values of m derived from (3.10) should correspond to degenerations of the curve.

A torus is defined by C/L, where L is a lattice in the complex plane generated by vectors $(1, \tau)$. We denote by z the coordinate parameterizing a fundamental cell in C/L. We represent again the orientifold action by $z \to -z$. There are four fixed points under this involution: 0, 1/2, $\tau/2$, $(\tau+1)/2$. Let us compactify the surface defined by the brane configuration by adding three points corresponding to the asymptotic regions $v \to \infty$ of each fivebrane. In analogy with the treatment of the genus zero curves, we want to construct now a holomorphic map from C/L to the type IIA ambient space

$$v = f_1(z) \quad , \; w = f_2(z) \quad , \; y = f_3(z) \ . \tag{4.3}$$

The functions $f_i(z)$ will be meromorphic and doubly periodic. We begin by determining f_1. The function f_1 should have three simple poles representing the positions of the fivebranes. Since one fivebranes is its own mirror under the orientifold action, one of the poles should be at an invariant point under $z \to -z$. Let us denote it by z_1, where z_1 will be 0, 1/2, $\tau/2$ or $(\tau+1)/2$; we call z_2, z_3, z_4 the other three invariant points. We set the other two poles at some points z_0, $-z_0$. A meromorphic function on a compact Riemann surface satisfies

$$\sum_j m_j = 0 \ , \tag{4.4}$$

where j labels the zeroes and poles of the function and m_j denotes its order at that points (1 if the function has a simple zero, -1 if it has a simple pole). Therefore f_1 will have in general three simple zeroes. We impose that they are at z_2, z_3, z_4. This determines the function f_1 up to a multiplicative constant.

There is however an important additional constraint. Since the variable v is odd under the orientifold involution, f_1 should be odd under $z \to -z$. In order to prove this we analyze $\tilde{f}_1 = f_1(z) + f_1(-z)$. $\tilde{f}_1$ can have only simple poles at $\pm z_0$, however it still has zeroes at z_2, z_3, z_4. The only possibility compatible with (4.4) is that $\tilde{f}_1 = 0$, and therefore the above defined function f_1 is odd.

The asymptotics as $v \to \infty$ tells us that f_2 should have two simple poles at $\pm z_0$, and that $w \sim \mu v$ at z_0 and $w \sim -\mu v$ at $-z_0$. This fixes f_2 up to shifts by a constant. Defining the function $\tilde{f}_2 = f_2(z) - f_2(-z)$ and using similar arguments to those applied to $\tilde{f}_1$, one can see that $\tilde{f}_2 = 0$. Thus f_2 is an even function under the orientifold involution, as needed in order to map it to w. At this point we could explicitly construct f_1 and f_2 in terms of the Weierstrass functions. The function f_3 will prove however to be more involved. Therefore we keep at a more abstract level, determining each function in terms of the singularity structure we desire but without attempting to give their explicit form.

On this line, we use both the asymptotic at $v \to \infty$ and $v = 0$ for determining f_3. Relations (4.1), (4.2) imply that f_3 must have a pole of order N_c at z_0, three poles of order 2 at z_2, z_3, z_4, a zero of order 2 at z_1 and a zero of order $N_c + 4$ at $-z_0$. These conditions fix f_3 up to a multiplicative constant. As it was the case with f_1 and f_2, f_3 should satisfy additional properties for correctly representing the coordinate y. First, invariance under the orientifold projection requires

$$f_3(-z)f_3(z)f_1^4(z) = \text{constant} . \tag{4.5}$$

This is indeed satisfied because the left hand side is a function without poles, and the only function without poles in a compact Riemann surface is a constant.

Next we have to check if f_3 fulfills (4.1), (4.2) with the indicated proportionality coefficients. Multiplying f_3 and f_1 by appropriate constants, we can set the constant in (4.5) to one and satisfy exactly conditions i) and iii). We have constructed the map (4.3) such that the asymptotics ii) in (4.1) and (4.2) appear at the four invariant points of the torus under $z \to -z$. At these points (4.5) reduces to

$$f_3^2 = v^{-4} . \tag{4.6}$$

Thus when $z \to z_i$, $i = 1, .., 4$, we get $y \to \epsilon v^{-2}$ with $\epsilon = \pm 1$. This is compatible with what we need, but still not a satisfactory answer. We need to obtain more information about the allowed distributions of ϵ values.

In order to proceed further we define the function $\tilde{f} = f_3 + f_1^{-2}$. This function will have a different singular behavior depending on the value of ϵ at each point z_i. When $\epsilon = -1$ at one of the z_i, the singularity order of $\tilde{f}$ at that point decreases with respect to that of f_3. One can see that the order $\tilde{f}$ at z_i must be odd if $\epsilon_{z_i} = -1$.[4] The function $\tilde{f}$ has a pole of order N_c at z_0 and a zero of order 2 at $-z_0$. It can also have zeroes at other points. Using again (4.5) we derive the following equality

$$\frac{f_3(z) + f_1^{-2}(z)}{f_3(-z) + f_1^{-2}(-z)} = f_1^2(z)f_3(z) . \tag{4.7}$$

This implies that the additional zeroes of $\tilde{f}$ are at paired points $\pm z_a$, with each pair having the same order. Using all this information we can see that consistency fixes the value of ϵ_{z_1} in terms of the values of ϵ at z_2, z_3, z_4. According to (4.2) we are interested

[4]For proving this we need to define still one more auxiliary function: f_3' such that $f_3 = f_3' f_1^{-2}$. At the invariant points $f_3' = \epsilon + az^n$. From (4.5) we get $f_3'(z)f_3'(-z) = 1$. This implies that n has to be an odd integer. Since f_1^{-2} is even under the orientifold involution, the order of $\tilde{f}$ at z_i is odd if $\epsilon_{z_i} = -1$.

in $\epsilon = -1$ at z_2, z_3, z_4. Applying (4.4) to $\tilde{f}$ implies that $\epsilon_{z_1} = (-1)^{N_c+1}$, in agreement with condition *ii*) in (4.1).

Finally, we will show that the distribution of ϵ values is correlated with the choice of point z_0. We will need to use an additional property satisfied by a meromorphic function on a torus

$$\sum_j m_j z_j = 0 \ (\text{mod } L) \ . \tag{4.8}$$

Applying (4.8) to f_3 restricts the allowed values of z_0 to those satisfying $(2N_c+4)z_0 = 0(\text{mod L})$. We see therefore that z_0 is not an additional modulus of our construction, but it is restricted to a discrete set of values. Applying (4.8) to $\tilde{f}$ constrains further z_0. As a result, $\epsilon = -1$ at z_2, z_3, z_4 is only compatible with $(N_c+2)z_0 = 0(\text{mod L})$ for N_c even and $(N_c+2)z_0 = z_1(\text{mod L})$ for N_c odd. Any other distribution of ϵ values implies different values for z_0.[5] Thus we can select the distribution of ϵ values compatible with the asymptotic behavior (4.1), (4.2) by choosing appropriately z_0.

We would like to comment on how to determine the elliptic modulus of the $\mathcal{N}=1$ curve. We could expand f_1 and f_3 around $z_1, \pm z_0$ and substitute in the $\mathcal{N}=2$ curve, as we did for the genus zero curves. In this way we would obtain relations between the coefficients of the subleading terms in f_3 and f_1 and the mass of the antisymmetric flavor. Since f_3, f_1 are fixed, these subleading coefficients are functions of the elliptic modulus τ and z_0. Therefore we will have relations between τ, z_0 and m, as we wanted. To be able to find a consistent set of relations for the subleading terms would be a further test of our genus one curve. This could restrict further the allowed values of z_0. The set of allowed z_0 should provide the $\mathcal{N}=1$ vacua of our theory.

The construction of genus one curve associated with the $\mathcal{N}=1$ $SU(N_c)$ theory with a massive antisymmetric flavor and $N_f=1$ can be done in a completely analogous way.[6] The functions $v = f_1(z)$ and $w = f_2(z)$ will have the same singularity structure as before. The function $y = f_3(z)$ should have now a pole of order N_c at z_0, a zero of order N_c+2 at $-z_0$, a simple zero at z_1 and simple poles at z_2, z_3, z_4. This function satisfies a version of (4.5)

$$f_3(z) f_3(-z) f_1^2(z) = \text{constant}. \tag{4.9}$$

From this one deduces $f_3 \to \epsilon v^{-1}$ when $z \to z_i$ with $\epsilon = \pm 1$. The $\mathcal{N}=2$ curve for $SU(N_c)$ with an antisymmetric flavor and $N_f = 1$ (2.12) reduces at $v=0$ to $(y+v^{-1})^2(y-v^{-1}) = 0$. We therefore set $\epsilon = -1$ at z_2, z_3, and $\epsilon = 1$ at z_4. It is convenient to define now $\tilde{f} = f_3 + f_1^{-1}$. Using the same considerations as previously one can see that the above values of ϵ at z_2, z_3, z_4 are only compatible with $\epsilon = (-1)^{N_c+1}$ at z_1. In agreement with this, the $\mathcal{N}=2$ curve at the pole associated with the central

[5] With the exception of $\epsilon = 1$ at z_2, z_3, z_4 and $\epsilon_{z_1} = (-1)^{N_c}$. These values just correspond to change $y \to -y$.

[6] We are referring again to a flavor induced by a pair of sixbranes placed over the orientifold sixplane, at $v = 0$.

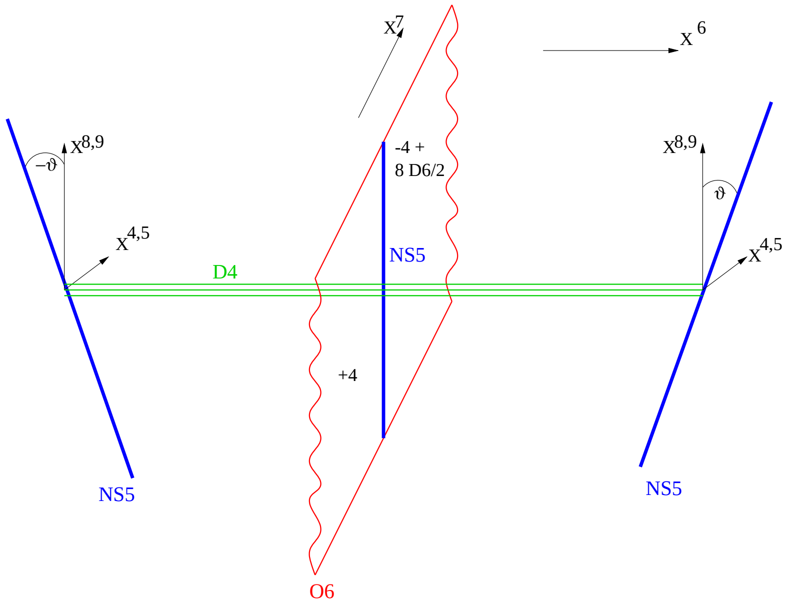

Figure 5: Brane configuration for the chiral theory.

fivebrane behaves as $y \to (-1)^{N_c+1} v^{-1}$. Finally, we can isolate as before the desired distribution of ϵ values by an appropriate choice of the point z_0.

5. Chiral theory

In this section we discuss the curve for the chiral theory with $SU(N_c)$ gauge group, $2N_f+8$ chiral multiplets in the fundamental representation, $2N_f$ in the antifundamental, one in the antisymmetric and one in the conjugate symmetric.

The brane construction that gives rise to this theory has previously been studied in [5]. It is again a configuration containing three fivebranes in the background of an orientifold sixplane. There are only two possible orientations for the middle fivebrane, which is its own mirror under the orientifold action. It can extend in the v-plane, as in the previous sections, or it can extend in the w-plane. This second possibility gives raise to the chiral theory. The orientifold sixplane is then divided in two by the middle fivebrane. At this point, the sign of the orientifold projection must change [20]. In order to compensate the change in Ramond charge of the orientifold, we have to introduce an additional set of 8 half D-sixbranes parallel to the orientifold plane, ending on the central fivebrane (see fig. 5). These give rise to the additional eight fundamentals [21]. Strings joining the sets of fourbranes to the left and right of the middle fivebrane induce now a chiral multiplet in the antisymmetric representation and an one in the conjugate symmetric representation.

We include also the presence of $2N_f$ sixbranes parallel to the orientifold sixplane. If the sixbranes are placed on top of the orientifold they get also cut by the middle fivebrane. In this special situation the matter content they induce on the gauge theory living on the fourbranes is doubled [5]. The flavor symmetry group is enhanced from $SU(N_f)_L \times SO(8)_L \times SU(N_f)_R$ to $SO(2N_f+8)_L \times Sp(2N_f)_R$. As before we can rotate the two outer fivebranes in a symmetric way with respect to the middle one by an angle θ. The tree-level superpotential for this brane configuration is

$$W = Q\tilde{X}_s Q + \tilde{Q} X_a \tilde{Q} + X_a X \tilde{X}_s + \mu X^2 \ , \tag{5.1}$$

where $\mu = \tan\theta$, X is the adjoint multiplet, $\tilde{X}_s$ the symmetric, X_a the antisymmetric, $\tilde{Q}$ the antifundamentals and Q the fundamentals.

The chiral brane configuration is such that the fourbranes to the left and right of the middle fivebrane can always be reconnected. Thus we expect that the associated curve factorizes into two pieces for any value of the parameters. The first piece will be a $\mathcal{P}^1$ representing the middle fivebrane as before. The second piece should describe a configuration with two fivebranes in a background space with a uniform +4 Ramond charge along the directions (0123789). Thus for $\mu = \infty$ this curve will coincide with that describing an $\mathcal{N} = 2$ $SO(N_c)$ theory with N_f massless flavors or equivalently, if N_c is even, an $Sp(N_c)$ theory with $N_f + 4$ massless flavors. When μ is finite, the curve for the chiral theory will coincide with the rotated curve for the SO or Sp theories with tensor matter mentioned above.

This chiral theory contains baryon operators $\tilde{B}_n = \tilde{X}_s^n \tilde{Q}^{N_c-n} \tilde{Q}^{N_c-n}$ and $B_n = X_a^n Q^{N_c-2n}$. By moving the central fivebrane in the positive x^7 direction we move onto the B_n baryonic branch and the gauge group breaks to $Sp(n)$ with $2N_f + 8 - 2(N_c - n)$ massless chiral fundamentals (with $SO(2N_f+8-2(N_c-n))$ flavor symmetry) [5]. Likewise, when we move the fivebrane in the negative x^7 direction, we move onto the $\tilde{B}_n$ baryonic branch and the gauge group breaks to $SO(n)$ with $2N_f - 2(N_c - n)$ massless fundamentals, with $Sp(2N_f - 2(N_c - n))$ flavor symmetry. The curve we have presented describes the chiral theory at the B_{N_c}, $\tilde{B}_{N_c}$ baryonic branch root. The other B_n, $\tilde{B}_n$ baryonic branches will be obtained by rotating the $\mathcal{N} = 2$ curve at the origin of the lower baryonic branches.

As in the $\mathcal{N} = 2$ case there is a $U(1)_X$, that now acts on X_a, $\tilde{X}_s$, Q and $\tilde{Q}$. Its Goldstone mode together with the x^7 position of the detached fivebrane gives rise to one complex modulus. This complex modulus appears to be frozen from the M-theory point of view since the fivebrane in question appears to be of infinite size. It is possible when one flows to the field theory limit, there are corrections to the Kahler potential, and the metric is no longer degenerate in this direction.

6. Higgsing for $\mathcal{N} = 2$

The Higgs branch appears when the M5-brane intersects a singular point in the multi-Taub NUT space. This point can be resolved into a number of rational curves which are then free to move off in the 7, 8, 9 directions [2], [14]. The Higgs moduli correspond to these parameters together with their superpartner, arising from integrating the chiral two-form of the worldvolume theory of the fivebrane over the rational curve.

6.1. Resolution of D_n singularity

We begin by reviewing the minimal resolution of a D_n singularity

$$a^2 + b^2 z = z^{n-1} \ . \tag{6.1}$$

The resolved surface is covered by n open sets $U_1, \cdots U_n$ with coordinates $(s_1, t_1, z_1) = (a, b, z/a)$, $(s_2 = b, t_2 = a/z, z_2), \cdots, (s_n, t_n, z_n)$. These are glued together via the transition relations

$$\begin{aligned}
(s_j, t_j, z_j) &= (s_{j+1} t_{j+1} z_{j+1}, s_{j+1}, t_{j+1}^{-1}) \quad j = 1, \cdots, n-4 \\
(s_{n-3}, t_{n-3}, z_{n-3}) &= (s_{n-2} t_{n-2}^2 z_{n-2}, s_{n-2} t_{n-2}, t_{n-2}^{-1}), \\
(s_{n-2}, t_{n-2}, z_{n-2}) &= (t_{n-1} z_{n-1}, s_{n-1}, t_{n-1}^{-1}) = (t_n^{-1}, s_n t_n, z_n) \ ,
\end{aligned} \tag{6.2}$$

and the projection to the a, b, z space is

$$\begin{aligned}
a &= s_{2j-1}^{j} z_{2j-1}^{j-1} &&= s_{2j}^{j} t_{2j} z_{2j}^{j} \\
b &= s_{2j-1}^{j-1} t_{2j-1} z_{2j-1}^{j-1} &&= s_{2j}^{j} z_{2j}^{j-1} \\
z &= s_{2j-1} z_{2j-1} &&= s_{2j} z_{2j} \ ,
\end{aligned} \tag{6.3}$$

for $U_1, \cdots, U_{n-3}, U_{n-1}$. For U_{n-2} and U_n we have

$$\begin{aligned}
a &= z^{n/2-2} s_{n-2} t_{n-2} &&= s_n z^{n/2-2}, && n: \text{ even} \\
b &= z^{[n/2]-1} t_{n-2} &&= s_n t_n z^{[n/2]-1}, && n: \text{ odd} \\
z &= s_{n-2} t_{n-2} z_{n-2} &&= s_n z_n \ .
\end{aligned} \tag{6.4}$$

The inverse image of the singular point consists of n rational curves C_i. For $i = 1, \cdots, n-2$ these are the z_i axis in U_i the t_{i+1} axis in U_{i+1}. C_{n-1} and C_n are the curves $t_{n-2} = z_{n-2} \mp 1 = 0$ in U_{n-2}. The D_n singularity in the i-th patch takes the form $s_i + t_i^2 z_i = s_i^{n-i-1} z_i^{n-i}$, $s_{n-2} + t_{n-2} z_{n-2} = s_{n-2} z_{n-2}^2$ in U_{n-2} and $1 + s_n t_n^2 z_n = z_n^2$ in U_n. Close to the exceptional divisor C_i we have $s_i, t_i \to 0$ and from the form of the D_n singularity in this patch $s_i \sim t_i^2$ while close to C_{i+1} $s_i, z_i \to 0$ with $s_i \sim z_i$. It is important to take the latter into consideration when counting the multiplicities of the C_i's on the Higgs branches.

6.2. $Sp(2N_c)$ gauge group

We consider the gauge group $Sp(2N_c)$ with N_f fundamental flavors. The singularity is of type D_{N_f}. The curve is given by $b = P(z)$ where $P(z) = z^{N_c} + u_2 z^{N_c - 1} + \cdots$. As long as $r \neq N_f/2$ (for the case N_f even), we can assume that at the r-th Higgs branch root

$$P(z) = z^r \tilde{P}(z) \, . \tag{6.5}$$

Away from $(a, b, z) = (0, 0, 0)$ we can rescale $(a, b) \to (z^r a, z^r b)$ and then we recover the curve for $Sp(2(N_c - r)) + N_f - 2r$ flavors, with $r \leq \min(N_c, [N_f/2])$. Close to the singularity the curve takes the form $b = z^r$. This equation has to be analyzed now in each of the coordinate patches defined above. On the rational curves $C_1, \cdots, C_{N_f}$ one finds $1, 2, \cdots, 2r-1, 2r, \cdots, 2r, r, r$ solutions respectively. In addition, one finds an infinite component that intersects the curve C_{2r}. From this it follows that the quaternionic dimension of the Higgs branch is given by $2rN_f - r(2r+1)$ in agreement with field theory. This has also already been discussed in a slightly different context in [13].

We have to be more careful when N_f is even and $r = N_f/2$. In this case we are left with the unbroken gauge group $Sp(2N_c - N_f)$ and no flavors. This curve does not live in a D_n space but in the Atiyah-Hitchin manifold

$$a^2 + b^2 z = 2b \,. \tag{6.6}$$

To go from D_{N_f} to Atiyah-Hitchin we have to redefine $(a, b) \to (z^{N_f/2}, z^{N_f/2-1}(bz-1))$. From this it follows that the polynomial $P(z)$ at the $N_f/2$-th Higgs branch root has to factorize as

$$P(z) = z^{N_f/2-1}(\tilde{P}(z)z - 1) \,. \tag{6.7}$$

We see that the location of this Higgs branch is shifted. It is rather nice to see how this shift appears through geometrical considerations by going from a D_{N_f} singularity to Atiyah-Hitchin. This constitutes another fine example for the interplay between geometry and gauge theory. In a different setup with an orientifold four-plane this shift has also found in [12]. In the odd coordinate patches the curve takes the form

$$s_{2j-1}^{j-1} z_{2j-1}^{j-1} \left(t_{2j-1} - s_{2j-1}^{N_f/2-j} z_{2j-1}^{N_f/2-j} (s_{2j-1} z_{2j-1} - 1) \right) = 0 \,, \tag{6.8}$$

for $j \leq N_f/2 - 2$ and in the $N_f - 2$-th patch it is

$$s_{N_f-2}^{N_f/2-1} t_{N_f-2}^{N_f/2-1} \left(1 - z_{N_f-2} (s_{N_f-2} t_{N_f-2} z_{N_f-2} - 1) \right) = 0 \,. \tag{6.9}$$

The exceptional divisors appear with multiplicity i for $i = 1 \cdots, N_f - 2$ with multiplicity $N_f/2 - 1$ for $i = N_f - 1$ and $N_f/2$ for $i = N_f$. All together this gives the correct dimension of the Higgs branch $N_f(N_f-1)/2$.

6.3. $SO(2N_c)$ gauge group

The gauge group $SO(2N_c)$ is obtained with a orientifold sixplane with $+4$ units of Ramond charge. The singularity is now of type D_{N_f+4}. This can be shown as in [8] by starting with the orientifold description as in (2.1) and then introducing invariant variables $a = v(x-y)$, $b = x + y$, $z = v^2$. We obtain then the D_{N_f+4} singularity with the additional restriction that we can only resolve down to D_4, which by itself represents the orientifold. The curve is given by $b = P(z)$. First consider the case where $SO(2N_c)$ with N_f flavors is broken down to $SO(2(N_c - r))$ with $N_f - 2r$ flavors. Near the singularity the curve is described by $b - z^r = 0$. We need to consider blow ups which leave a D_4 singularity intact. This corresponds to blowing up only the first N_f rational curves that appear in the resolution of a D_{N_f+4} singularity. The multiplicities are identical to those found above, namely $1, 2, \cdots, 2r-1$ for $C_1, \cdots, C_{2r-1}$ and $2r$ for $C_{2r}, \cdots C_{N_f}$. Summing the total number of rational curves gives $2rN_f - 2r^2 + r$ for the quaternionic dimension of the Higgs branch, in agreement with field theory.

It is also possible to consider Higgsing $SO(2N_c)$ with N_f flavors down to $SO(2N_c - 2r - 1)$ with $N_f - 2r - 1$ flavors. This can be understood as a sub-branch of the previous case, except when N_f is odd and $r = [N_f/2]$. Now we assume that $P(z) = z^{r+1}\tilde{P}(z)$. To obtain the curve away from the singularity we rescale $b \to z^r a$ and $a \to z^{r+1} b$. These rescalings follow from demanding that we have a D_4 singularity $zb^2 + a^2 = z^3$ remaining. Note that the curve is described by $a = z\tilde{P}(z)$. Relative to the previous case, the roles played by the coordinates a and b are interchanged. Curves for $SO(2N_c)$ are described by $b = P(z)$ in a D_{N_f+4} space (6.1) whereas curves for $SO(2N_c+1)$ are described by $a = zP(z)$ with P being a polynomial of order N_c. We will explain this immediately in the next subsection. The curve near the singularity takes the form $a - z^{1+r} = 0$. Analyzing this on the patches $U_1, \cdots, U_{N_f}$ one finds the solutions on the curves $C_1, \cdots, C_{N_f}$ with multiplicities $2, 3, \cdots, 2r+1, 2r+2, \cdots 2r+2$. On each of these C_i one of these solutions corresponds to an additional infinite D-fourbrane along the z axis. To Higgs to $SO(2N_c - 2r + 1)$ we want to keep this D-fourbrane intact. The quaternionic dimension of the moduli space is then $\sum_{i=1}^{2r+1} i + (2r+1)(N_f - 2r - 1) = (2r+1)(N_f - r)$.

6.4. $SO(2N_c+1)$ gauge group

Our first task is now to understand the particular form for the curves. We start with the description in terms of x, y and v,

$$xy = v^{4+2N_f} \qquad y^2 + yvP(v^2) - v^{4+2N_f} = 0\,. \tag{6.10}$$

This can be written as $y - x = vP(v^2)$. In order to write this in terms of invariant variables we have to multiply with an overall factor of v. This means that we add an additional infinite fourbrane at $v = 0$. Now the curve can be written in a D_{N_f+4} space as $a = zP(z)$. This explains the form that we obtained by Higgsing previously. Assuming $P(z) = z^r\tilde{P}(z)$ we get the curve away from the singularity by taking $(a, b) \to (z^{r+1}b, z^r a)$ and then we recover the curve for $SO(2N_c - 2r)$ gauge theory with $N_f - 2r - 1$ flavors.

We can now go on and count the multiplicities of the exceptional divisors in the blown up space. Close to the singularity the curve takes the form $a = z^{r+1}$. In the i-th patch we find the exceptional divisor with multiplicity $i+1$ for $i = 1, \cdots, 2r$ and with multiplicity $2r+2$ for $i = 2r+1, \cdots, N_f$. Now we have to remember that we had to add an additional fourbrane to write the curve in invariant variables. This fourbrane does not contribute to the Higgs branch of the gauge theory. However, we still should expect to see it in all the patches. Thus in order to compute the dimension we should subtract one from the multiplicities in each patch. We get then $N_f(2r+1)-r(2r+1)$ which indeed is the correct quaternionic dimension of the Higgs branch.

The cases with unbroken gauge group $SO(2(N_c-r)+1)$ and N_f-2r flavors can be understood as sub-branches of the previous case except for N_f even and $r=N_f/2$, when we break to the theory with no flavors. We assume $P(z)=z^{N_f/2}\tilde{P}(z)$. To get the curve away from the singularity we have to rescale $(a,b)\to(z^{N_f/2}a, z^{N_f/2}b)$. Close to the singularity the curve is again $a=z^{N_f/2+1}$. Counting the multiplicities in the same way as before we get for the dimension of the Higgs branch $N_f(N_f+1)/2$.

7. Higgsing for $\mathcal{N}=1$

The Higgs branch for $\mathcal{N}=2$ is typically only a subspace of the full Higgs branch of the $\mathcal{N}=1$ SO/Sp theory with tensors. The most interesting case to consider is when we send $\mu\to\infty$. The outer NS-fivebranes are parallel to the orientifold sixplane, and new moduli open up corresponding to fourbranes moving along the outer fivebranes. Additional moduli also arise from an infinite component of the $\mathcal{N}=1$ curve degenerating as $\mu\to\infty$ into another infinite curve plus rational curves.

7.1. $Sp(2N_c)$ gauge group

For $\mu\to\infty$ we have an $Sp(2N_c)$ gauge theory with a massless antisymmetric tensor and N_f flavors of fundamental. The limit is taken by rescaling $\tilde{y}=\mu^{2N_c}y$, $\tilde{x}=\mu^{2N_c}x$. The scale $\Lambda_{\mathcal{N}=1}$ is held fixed in this limit, and is related to the $\mathcal{N}=2$ scale by

$$\mu\Lambda_{\mathcal{N}=2}=(\Lambda_{\mathcal{N}=1}^{4N_c+8-2N_f}\mu^{-2N_f+4})^{\frac{1}{4N_c+4-2N_f}}\ . \tag{7.1}$$

Note that as we send $\mu\to\infty$ we not only integrate out the massive adjoint chiral multiplet of the $\mathcal{N}=2$ theory, but we integrate in a light antisymmetric tensor with mass $1/\mu$. This spoils the naive dimension counting in (7.1). There are three cases to consider, $N_f<2$, $N_f=2$ and $N_f>2$. In the first case, $\mu\Lambda_{\mathcal{N}=2}$ blows up as $\mu\to\infty$ and the curve becomes infinitely stretched in the x^6 direction. This is the manifestation in M-theory of the fact that the field theory has a runaway vacuum state [22].

For $N_f>2$, $\mu\Lambda_{\mathcal{N}=2}$ vanishes as $\mu\to\infty$ and the $\mathcal{N}=1$ curve splits into an infinite component

$$C:\quad v=0,\quad \tilde{x}=0,\quad \tilde{y}=w^{2N_c}\ , \tag{7.2}$$

and its $\mathbb{Z}_2$ symmetric partner, where $\tilde{x}$ and $\tilde{y}$ are interchanged. In addition, there are a number of rational curves. These arise from the same scaling limit as C, namely $\mu\to\infty$, with $w^2\sim(\mu v)^2$ held finite.

We wish to analyze the equations for the curve in the resolved D_{N_f} space as $\mu\to\infty$. Up to trivial rescalings, these equations reduce to

$$a=v(\tilde{y}-\tilde{x})=w^{2N_c+1}/\mu$$

$$
\begin{aligned}
b &= (\tilde{x} + \tilde{y}) = w^{2N_c} \\
z &= -(w/\mu)^2 \ .
\end{aligned}
\tag{7.3}
$$

On the U_{2j} patch this yields

$$
\begin{aligned}
s_{2j} &= \mu^{2j-2} w^{2N_c-2j+2} \\
z_{2j} &= \mu^{-2j} w^{2j-2N_c} \ .
\end{aligned}
\tag{7.4}
$$

For $j > N_c$ these equations can never be satisfied for finite w and s_{2j} so the curve does not have a solution in this patch. For $j \leq N_c$, we find solutions with $2(N_c - j)$ multiplicity on the curve C_{2j}. Likewise $2(N_c - j) + 1$ solutions are found on C_{2j-1}. The total number of additional rational curves that appear is $\sum_{i=1}^{2N_c}(2N_c - j) = 2N_c^2 - N_c$. This gives rise to $4N_c^2 - 2N_c$ complex moduli.

In addition, there are an extra $2N_c$ complex moduli arising from the blowup of the infinite component C to $\tilde{y} = w^{2N_c} + a_1 w^{2N_c-1} + \cdots + a_{2N_c}$. The total number of complex moduli is then that of the $\mathcal{N} = 2$ theory, plus the contributions from these two other sources. This totals $4N_f N_c - 2N_c$, in agreement with the field theory result [17].

For the special case $N_f = 2$ a similar calculation leads to the same result for the dimension of the Higgs branch. In this case, $\mu\Lambda_{\mathcal{N}=2}$ is held finite and the form of the curve in the $\mu \to \infty$ limit differs from (7.2).

7.2. $SO(2N_c)$ gauge group

The $SO(2N_c)$ case may be treated in a similar way. Now the scale $\Lambda_{\mathcal{N}=1}$ is related to the $\mathcal{N} = 2$ scale by

$$
\mu\Lambda_{\mathcal{N}=2} = (\Lambda_{\mathcal{N}=1}^{4N_c-8-2N_f} \mu^{-2N_f-4})^{\frac{1}{4N_c-4-2N_f}} \ ,
\tag{7.5}
$$

In the $\mu \to \infty$ limit $\mu\Lambda_{\mathcal{N}=2}$ vanishes and the $\mathcal{N} = 1$ curve splits into an infinite component

$$
C: \quad v = 0, \quad \tilde{x} = 0, \quad \tilde{y} = w^{2N_c} \ ,
\tag{7.6}
$$

an its $\mathbb{Z}_2$ image with $\tilde{x}$ and $\tilde{y}$ interchanged. There is also a number of rational curves, which arise from the same scaling limit as C; $\mu \to \infty$, with $w^2 \sim (\mu v)^2$ held finite.

The counting of the rational curves proceeds as in the previous subsection, with the only difference being that now the curve sits in the resolved D_{N_f+4} space. $4N_c^2 - 2N_c$ extra complex moduli come from these curves, and $2N_c$ come from the blow up of the infinite component C. The total number of complex moduli for the Higgs branch is therefore $2(2N_c N_f - 2N_c^2 + N_c)$ from the $\mathcal{N} = 2$ branch plus these additional contributions, which totals $4N_c N_f + 2N_c$. This agrees with the field theory result.

7.3. $SO(2N_c + 1)$ gauge group

The $\mu \to \infty$ limit of the curve takes the form

$$
C: \quad v = 0, \quad \tilde{x} = 0, \quad \tilde{y} = w^{2N_c+1} \ ,
\tag{7.7}
$$

plus its $\mathbb{Z}_2$ image, plus rational curves, which arise from the same scaling limit as C, $w^2 \sim (\mu v)^2$.

To write the curve in a, b, z coordinates of D_{N_f+4} space, we add an extra infinite D-fourbrane by multiplying through by an extra power of v as explained in the subsection 6.4. The curve as $\mu \to \infty$ is taken to be

$$\begin{aligned} a &= v(\tilde{y} - \tilde{x}) = w^{2N_c+2} \\ b &= (\tilde{x} + \tilde{y}) = \mu w^{2N_c+1} \\ z &= -(w/\mu)^2 \ . \end{aligned} \tag{7.8}$$

Analyzing these equations in the U_{2j} patch, one finds $2j$ solutions on the C_{2j-1} curve for $2j < 2N_c + 2$. On C_{2j} one finds $2j + 1$ solutions for $2j < 2N_c + 1$. One of these solutions on each C_i corresponds to the additional infinite D-fourbrane along the z-axis, which appeared in the $\mathcal{N} = 2$ analysis. The number of additional rational curves is therefore $\sum_{i=1}^{2N_c} i = N_c(2N_c+1)$. There are also $2N_c+1$ complex moduli arising from the blowup of the infinite component C. The total number of additional complex moduli is $(2N_c + 1)^2$, which when combined with the moduli of the $\mathcal{N} = 2$ analysis, gives $2(2N_c + 1)N_f + (2N_c + 1)$. This agrees with the field theory analysis.

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